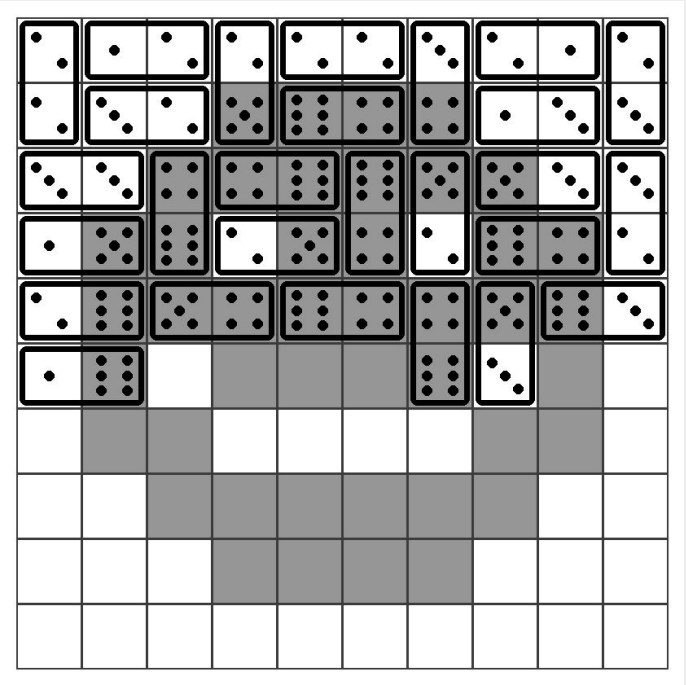
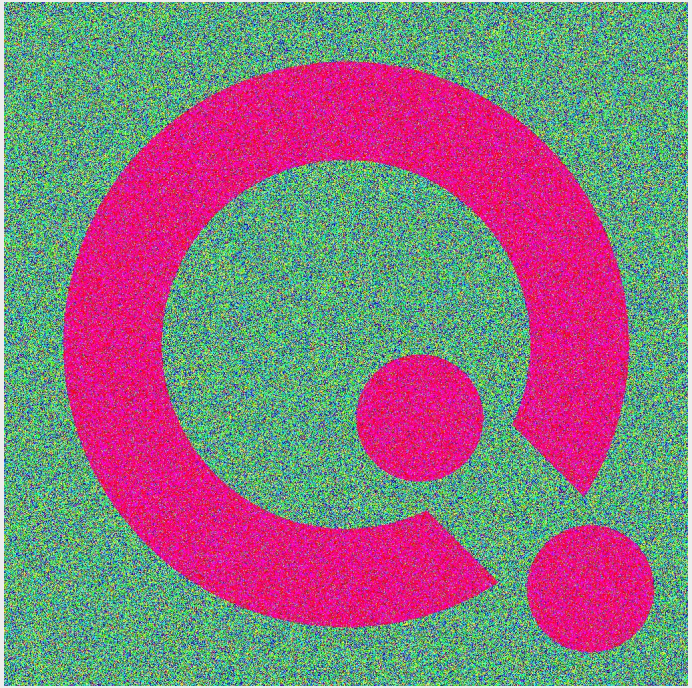
Domino tiling problem
CTU 2026

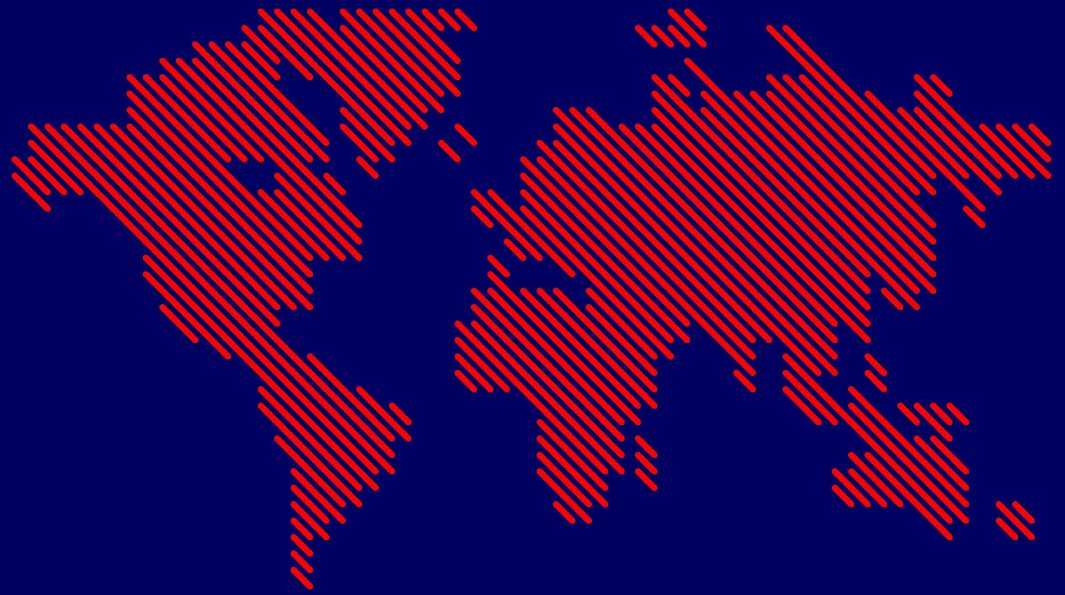


I. Problem statement





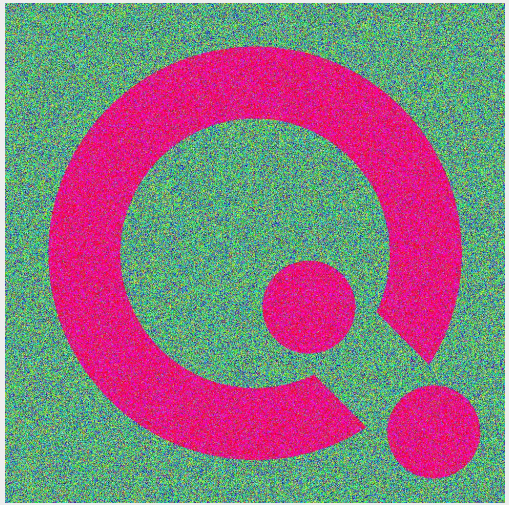
1. Heuristic approaches

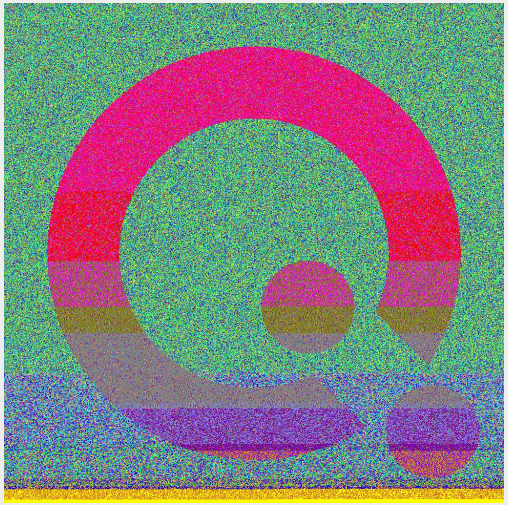


II. Greedy solution

Lay all tiles horizontally, top dow, left to right, always picking the most suitable available one.

Cost: ~199 589, Complexity: O(M·N) (we consider the amount of unique tiles to be constant)



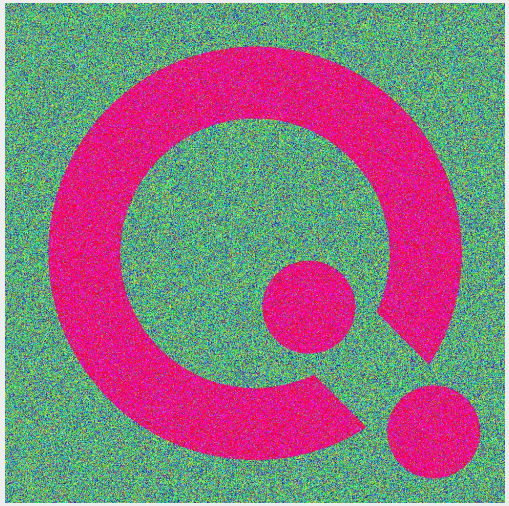


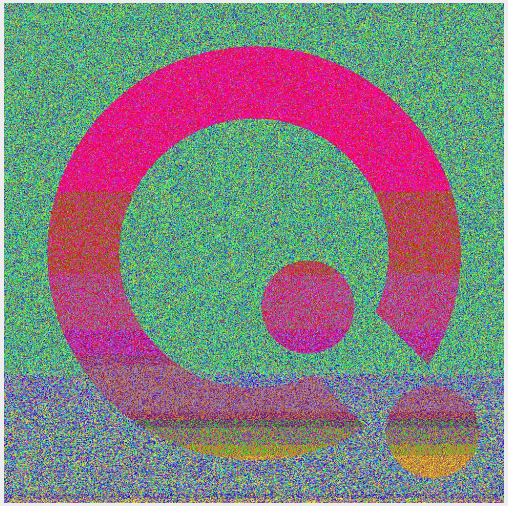


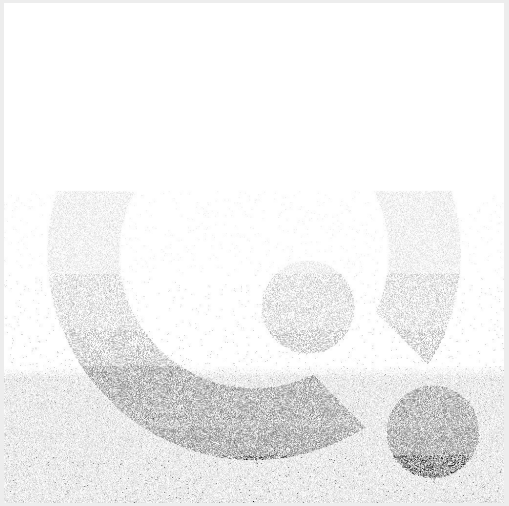
III. Greedy solution 2

Lay all tiles horizontally, starting by the best possible matches.

Cost: ~160 229, Complexity: O(M·N)



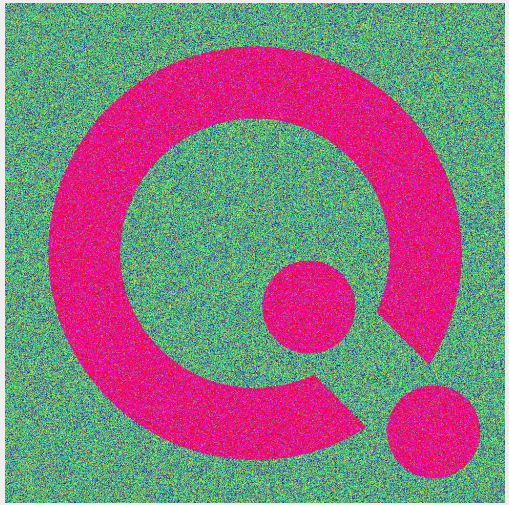


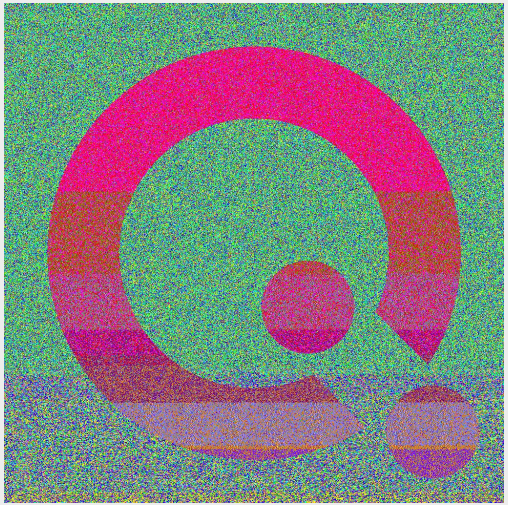


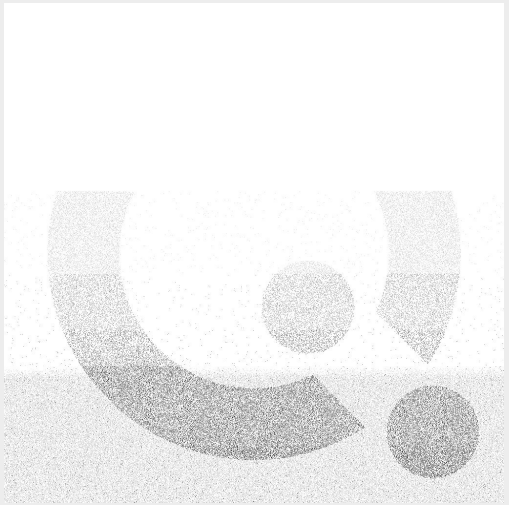
IV. Flow based solution

Formulate the problem as flow. Still only horizontal tiles.

Cost: ~154 794, Complexity: O(M·N)



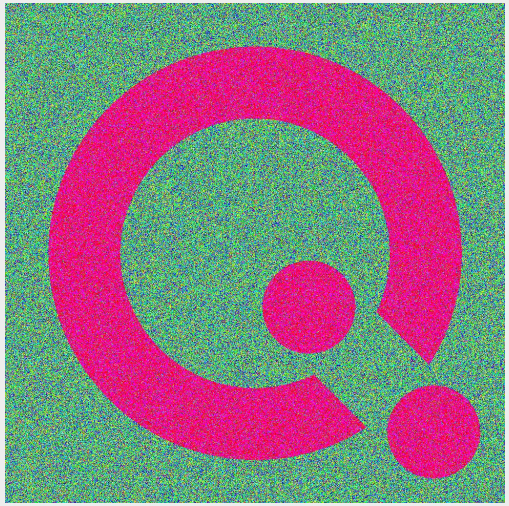


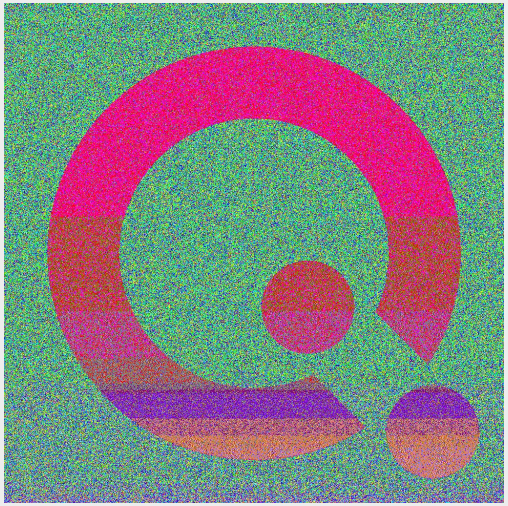


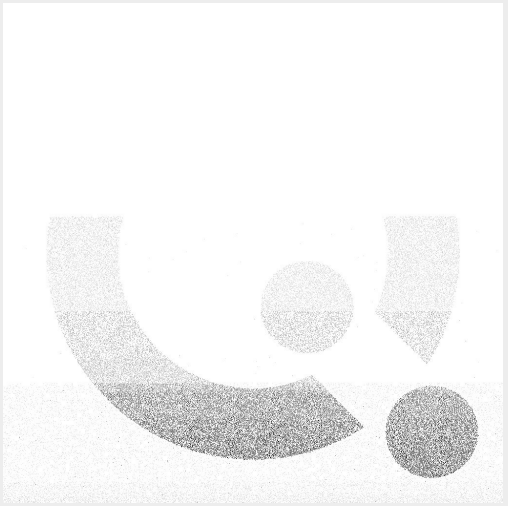
V. Flow based solution with flips

Same as previous, except try rotating 2x2 squares in hopes of finding better solution.

Cost: ~98 188, Complexity: $O((M \cdot N)^2 \cdot \log (M \cdot N))$







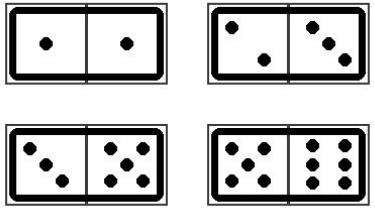
VI. Flow based solution with flips

Non-optimality proof:

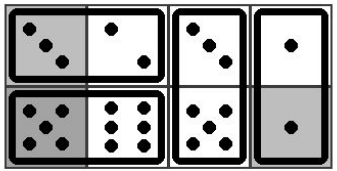
Design

4	2	3	1
3	6	5	2

Dominoes

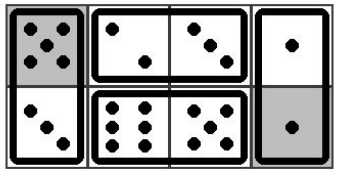


What the algorithm finds



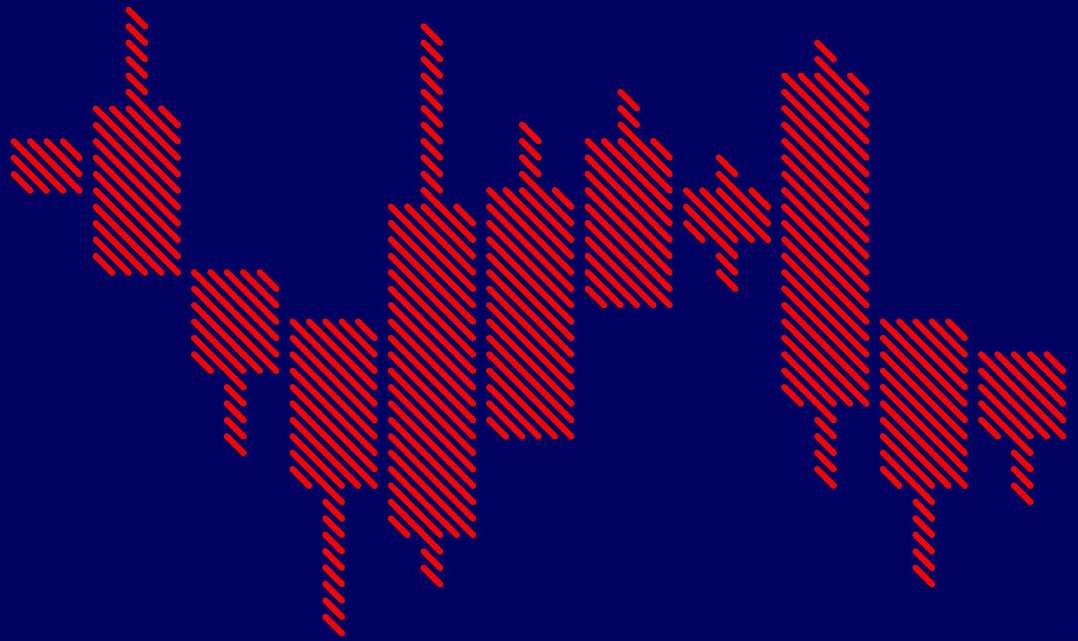
Total cost 2 + √2 ≈ 3.414

Optimal solution

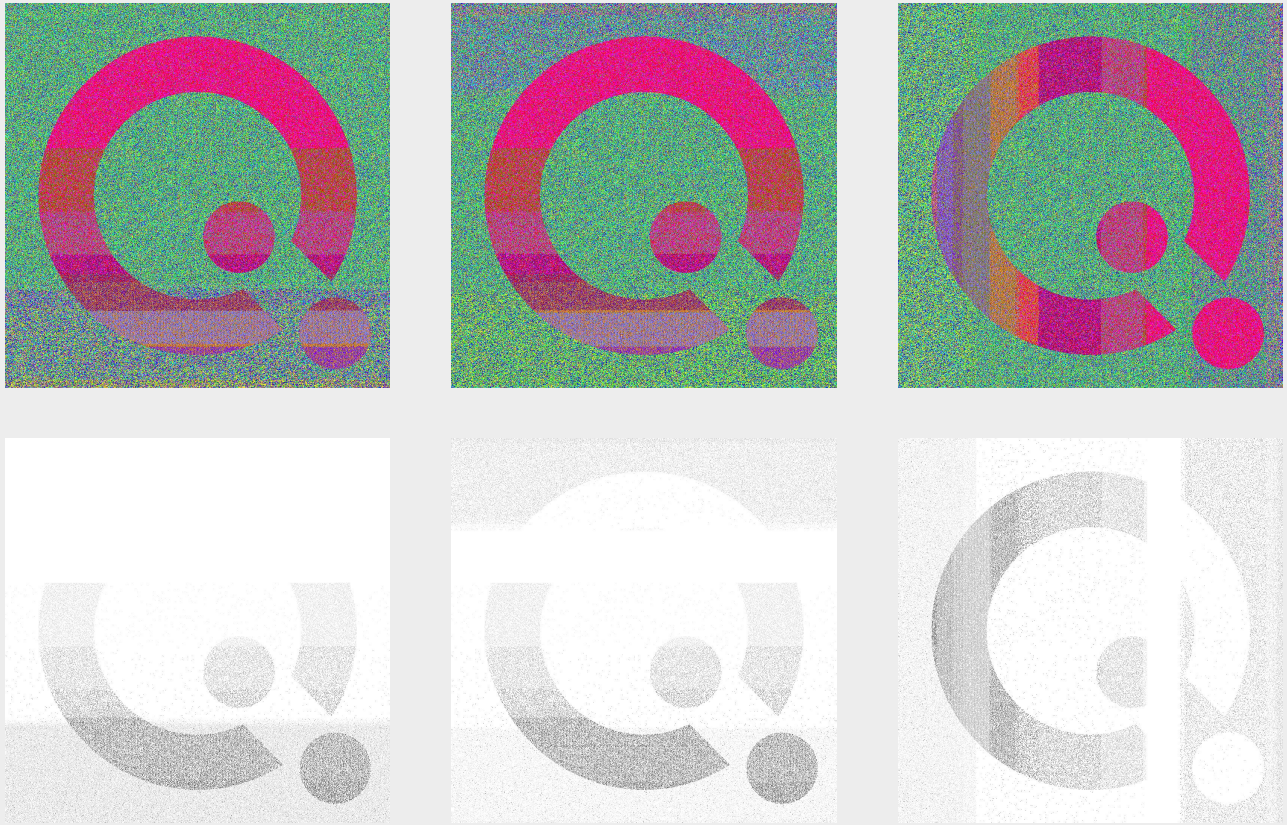


Total cost 2

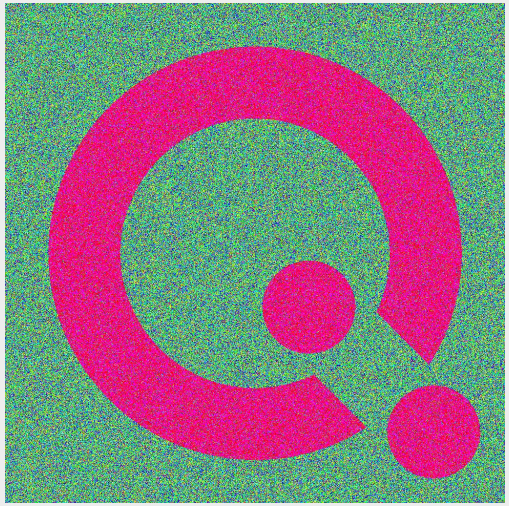
2. Solutions from you

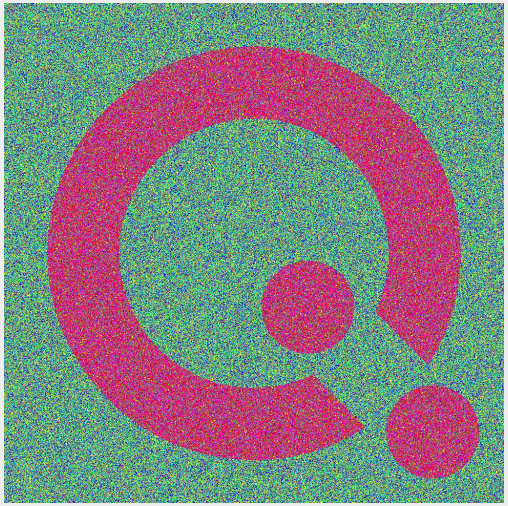


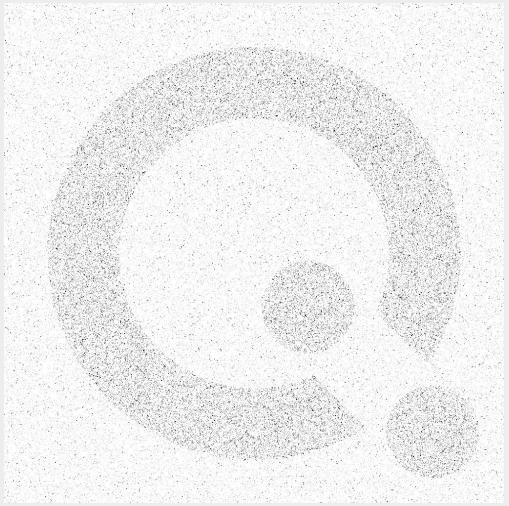
VII. Flow based solutions



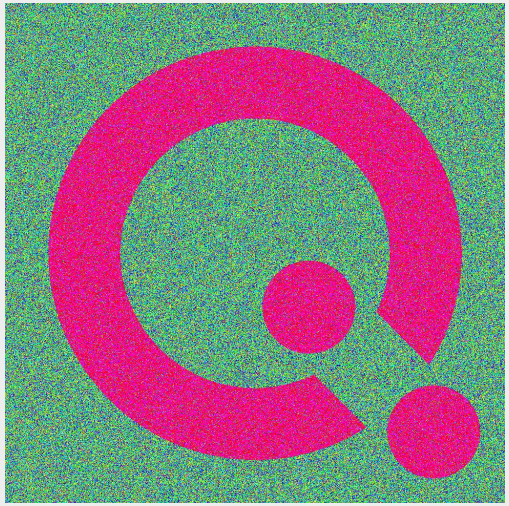
VIII. Greedy 2x2 + local search

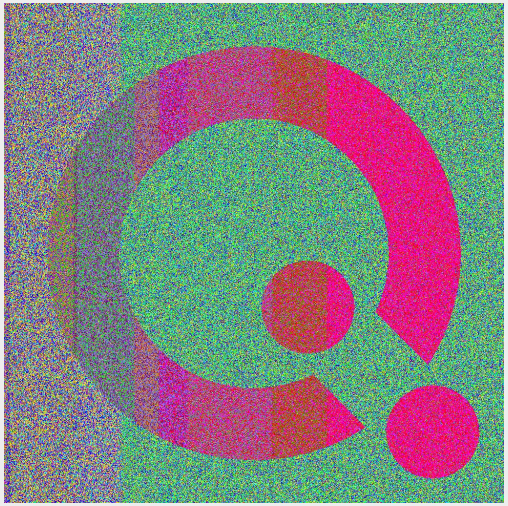


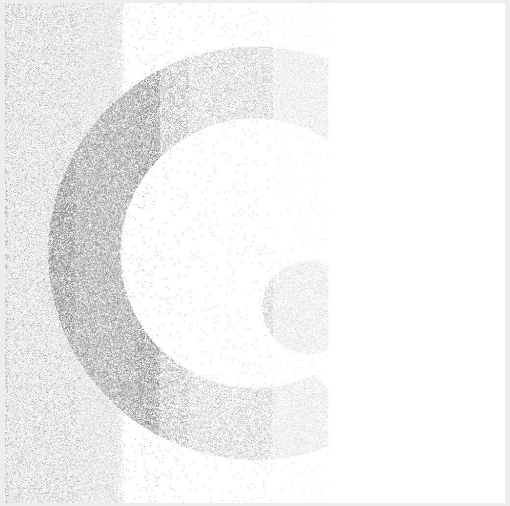




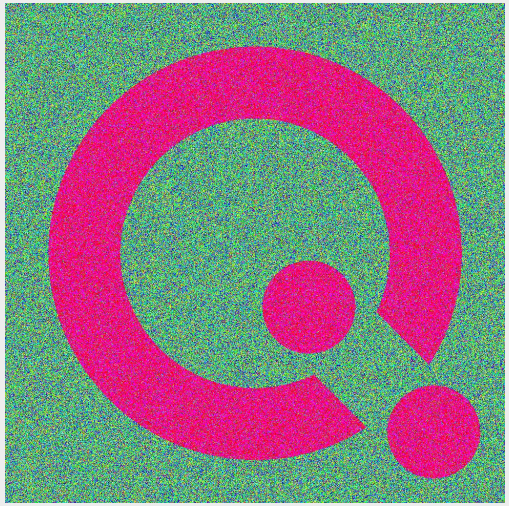
IX. Greedy 2 except vertical

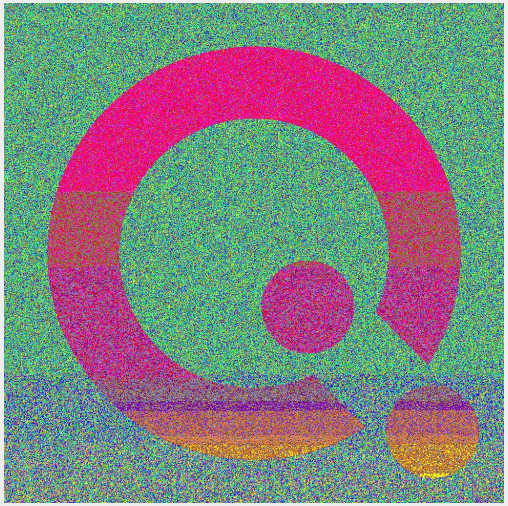


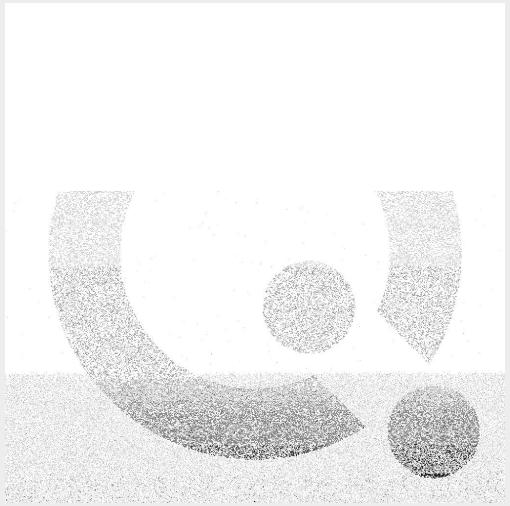




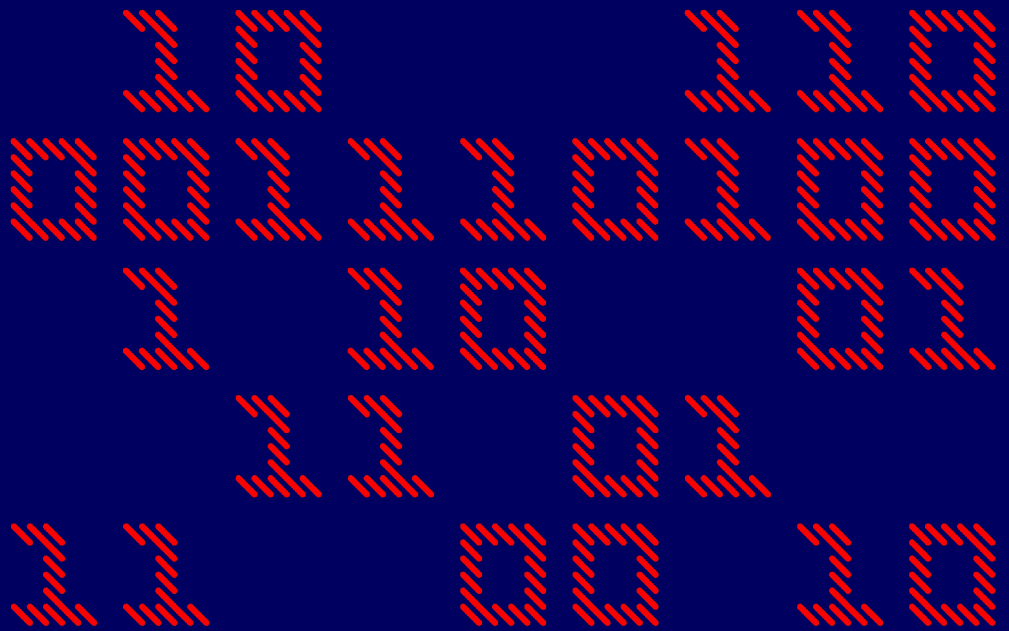
X. Greedy 2 except on 2x2 blocks







3. Exact polynomial solution



XI. Exact polynomial solution

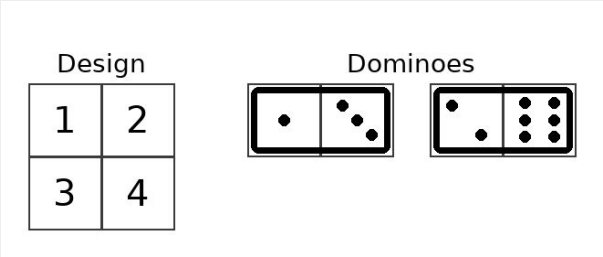
Note that this solution is not practically usable, just a proof that the problem is in P

1. Formulate the existential problem as matrix permanent calculation
   - each domino type gets its own variable, ex. $x_{(1,2)}$
   - each possible cost also gets its own variable, ex. $c_{(1,2)}$
   - color in cells of design checkerboard style
   - make a matrix, whose columns correspond to white cells and rows to black cells
   - fill in its cells by polynomials
     - 0 if the corresponding cells aren't adjacent
     - otherwise construct polynomial. The example below corresponds to case, where a (2,3) slot can be filled by either (1,2) or (5,5) dominoes.

$$x_{(1,2)}c_{(1,1)} + x_{(5,5)}c_{(2,3)}$$

2. Calculate determinant instead (see Kasteleyn's Theorem)
3. Try a domino placement and iterate

XII. Exact polynomial solution

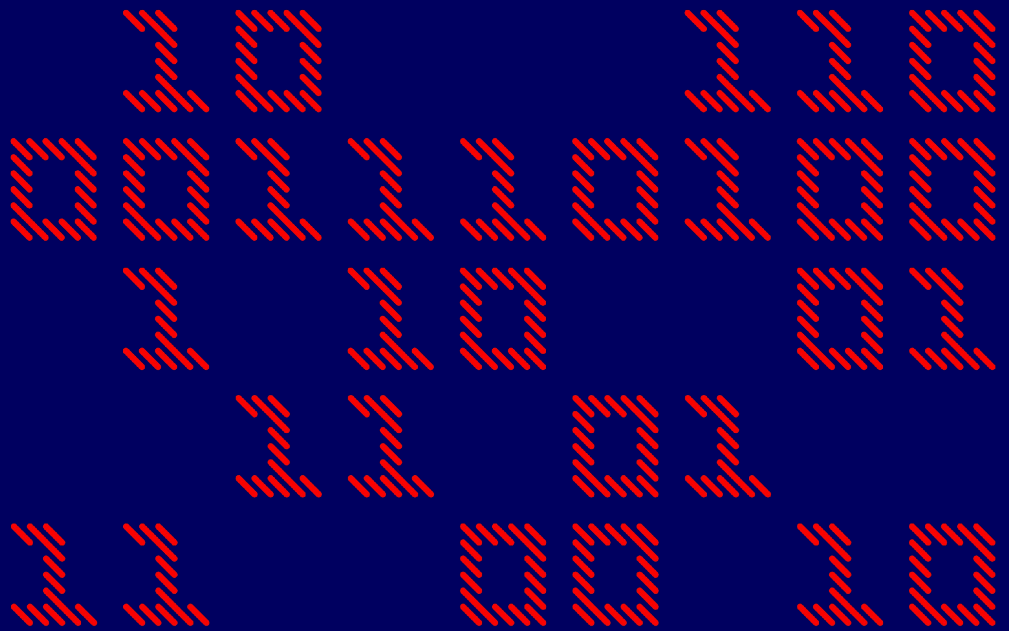


$$\begin{pmatrix} x_{(1,3)}c_{(0,1)} + x_{(2,6)}c_{(0,5)} & x_{(1,3)}c_{(0,0)} + x_{(2,6)}c_{(1,3)} \\ x_{(1,3)}c_{(1,1)} + x_{(2,6)}c_{(0,2)} & x_{(1,3)}c_{(0,3)} + x_{(2,6)}c_{(1,2)} \end{pmatrix}$$

$$x_{(1,3)}^2\, c_{(0,1)}c_{(0,3)} + x_{(1,3)}x_{(2,6)}\, c_{(0,1)}c_{(1,2)} + x_{(1,3)}x_{(2,6)}\, c_{(0,5)}c_{(0,3)} + x_{(2,6)}^2\, c_{(0,5)}c_{(1,2)} + x_{(1,3)}^2\, c_{(0,0)}c_{(1,1)} + x_{(1,3)}x_{(2,6)}\, c_{(0,0)}c_{(0,2)} + x_{(1,3)}x_{(2,6)}\, c_{(1,3)}c_{(1,1)} + x_{(2,6)}^2\, c_{(1,3)}c_{(0,2)}$$

$$x_{(1,3)}x_{(2,6)}\, c_{(0,1)}c_{(1,2)} + x_{(1,3)}x_{(2,6)}\, c_{(0,5)}c_{(0,3)} + x_{(1,3)}x_{(2,6)}\, c_{(0,0)}c_{(0,2)} + x_{(1,3)}x_{(2,6)}\, c_{(1,3)}c_{(1,1)}$$

4. Scores



XIII. Scores

#	Team name	cteam#	Cost
1	Dejvice, dej více!	cteam35	149,938
2	Teletubbies	cteam16	162,748
3	Lambda mačka	cteam55	163,852
4	MFF1	cteam24	165,468
5	Začiatok koncov	cteam61	172,102
6	MetalPipeFalling	cteam26	194,659
7	(blank)	cteam58	437,358
8	ICPC-larpers	cteam06	512,566
9	BuZa	cteam03	646,206
10	mo[d|k]re tricko	cteam71	950,525
11	Div 0	cteam20	1,018,505
12	std::back_inserter	cteam57	1,295,538
13	make no mistakes	cteam25	1,320,812
14	cteam67	cteam41	1,322,748
14	VÝPOČETNÍ A APLIKOVANÁ MATEMATIKA FEI VŠB-TUO - TÝM ZA OPTIMALIZACI ALGORITMŮ, MINIMALIZACI ČASOVÉ SLOŽITOSTI, MAXIMALIZACI BODOVÉHO ZISKU, DOKONALÉ ŘEŠENÍ ÚLOH A BEZCHYBNÝ KÓD!	cteam82	1,322,748
14	O(eventually)	cteam85	1,322,748
17	Nejmenší číslo, které není možné žádným způsobem definovat pomocí českého slovního útvaru obsahující počet slov ostře menší než dvacet jedna	cteam22	1,444,091

Thank you
for watching

