CTU Open 2024

Presentation of solutions

October 19, 2024

Flagbearer

Bear

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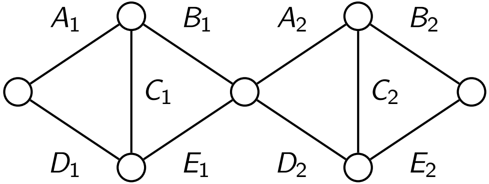
- You can also observe a regular pattern in the rotation of the hands (with few exceptions).

Fellow Sheep

Sheep



- The task boils down to finding the maximum flow between the pasture, and the farmyard.

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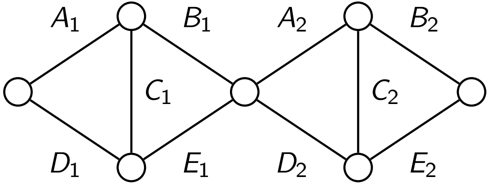

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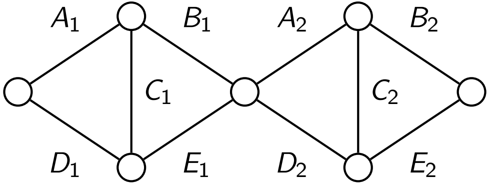

- The task boils down to finding the maximum flow between the pasture, and the farmyard.
- Maximum flow is equal to the minimum cut.
- Observation: for each segment, we can find the minimum cut separately.

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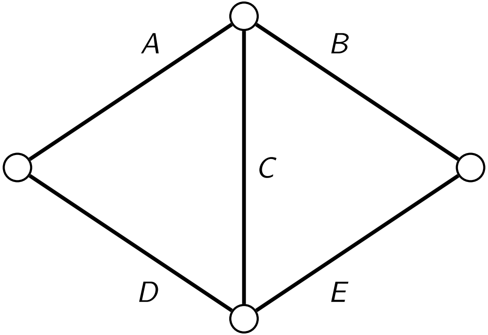

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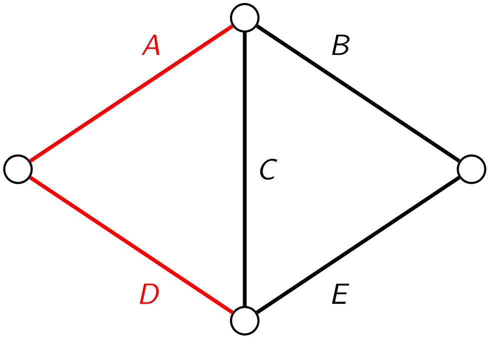

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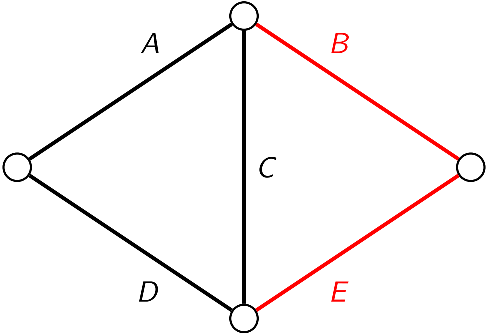

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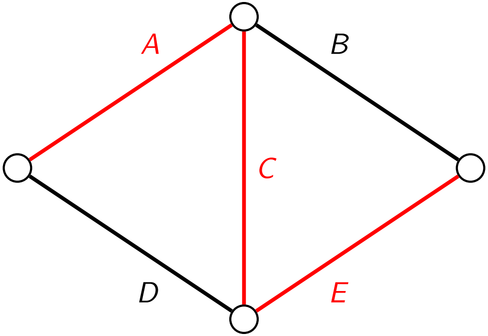

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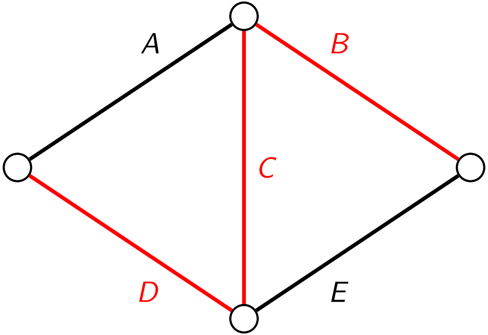

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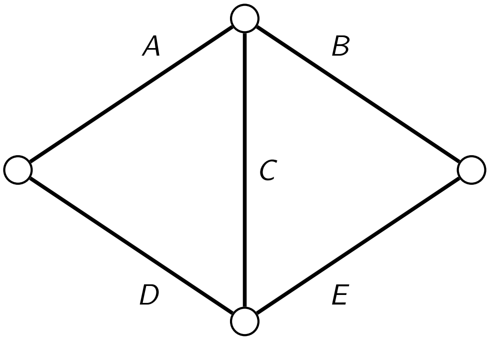

$$min(A + D, B + E, A + C + E, B + C + D)$$

Sheep

Repeat for each segment, take the minimum of all segments.

Hamster

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- **Input:** $N \times M$ grid $P_N \times P_M$, each cell with nonegative integer.
- **Output:** Find a maximum cost path from top left to bottom right.

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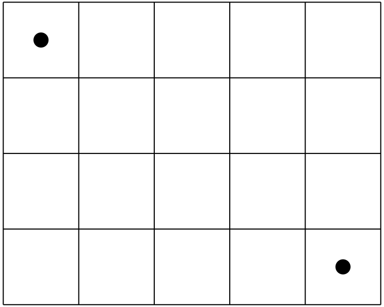

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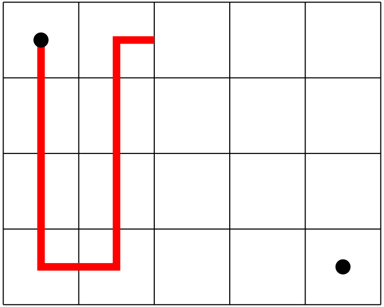

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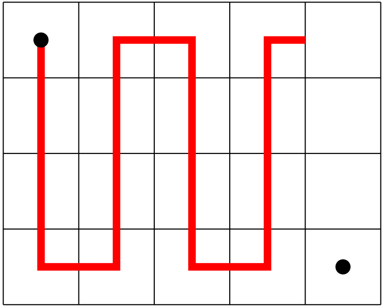

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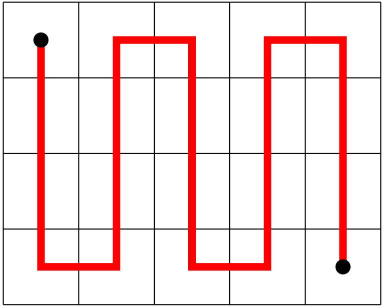

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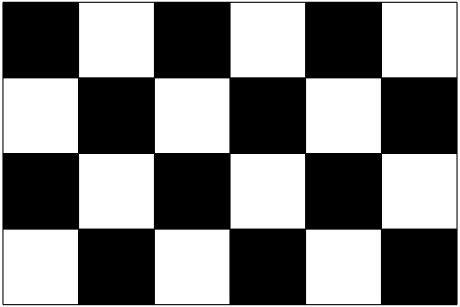

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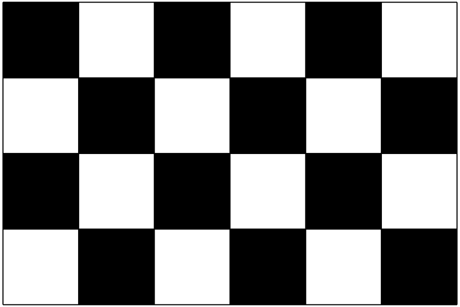

Colorally: Each path contains one more black than white cell.

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Observation 3: For each white cell, there is a path that only removes that cell.

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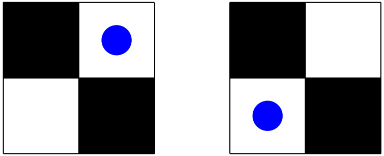
- **Base case:**



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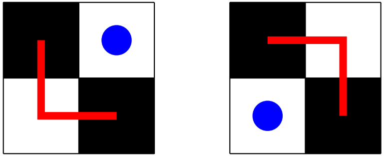

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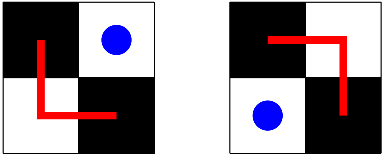
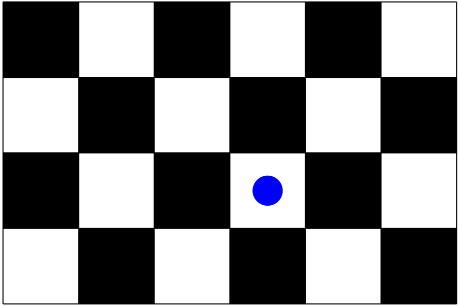


- **Induction step:** Remove 2 left/most most column or 2 first/last rows that do not contain the cell in question.



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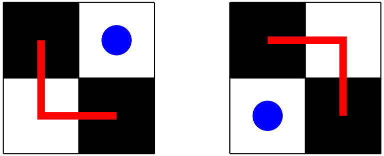

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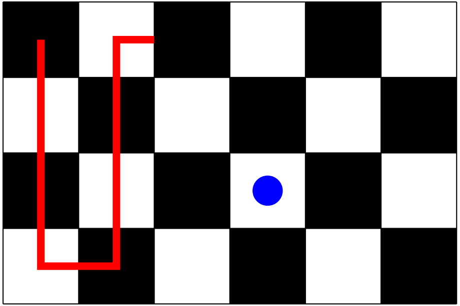

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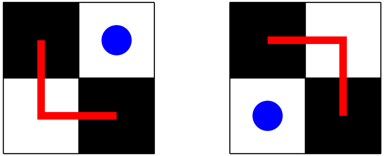

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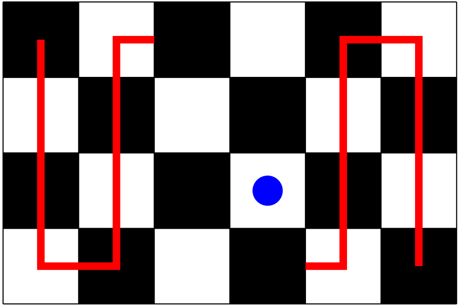

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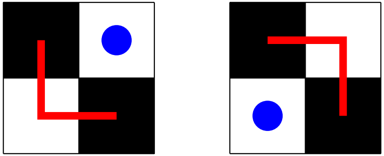

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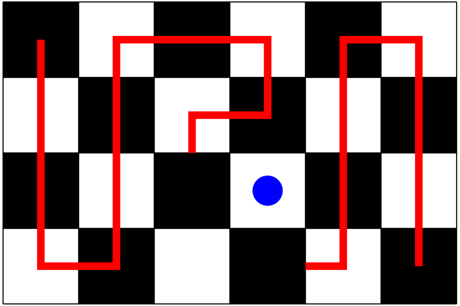

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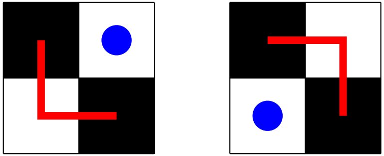

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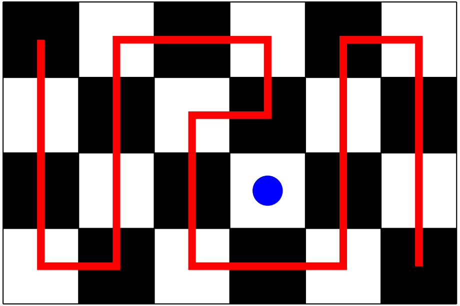


Hamster

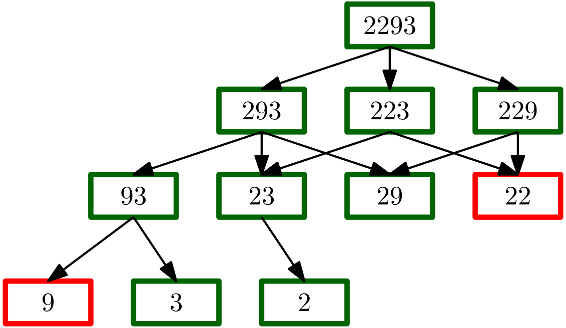
Summary:

- If N or M is odd, return sum of all cells.
- Else both N and M are even.
  - Let C_{MIN} be the minimum value on a cell that has sum of its coordinates odd.
  - Output sum of all cells minus C_{MIN}.

Pray mink

Mink

- **Input**: Integer $1 \leq N \leq 10^9$.
- **Task**: How many prime numbers can we obtain in a row when removing digits from N one by one?



Mink

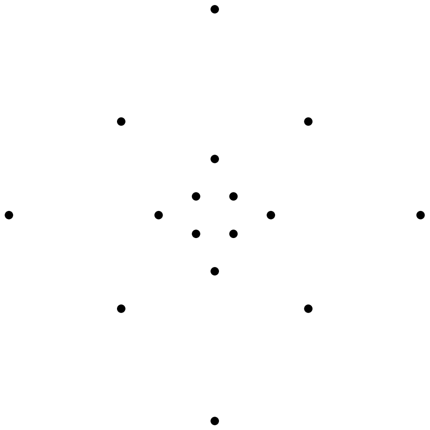
- N has at most 10 digits.
- We can obtain 2^{10} different numbers.
- We can recursively try all possible sequences of removing numbers and use DP.
- Checking primeness in time $O(\sqrt{n})$ is fast enough.
  - At most $\sum_{i=1}^{10} \binom{10}{i} \sqrt{10^i} \approx 1.6 \cdot 10^6$ modulo operations.
- The rest takes $O(2^{\log n}) = O(n)$ time.

$$ \tag{1} $$

Fishception

Fish

- $4N$ points with integer coordinates in a plane.
- The points represent the vertices of non-intersecting rectangles, each one contained within the next.
- **Task:** Determine the area of the smallest rectangle.



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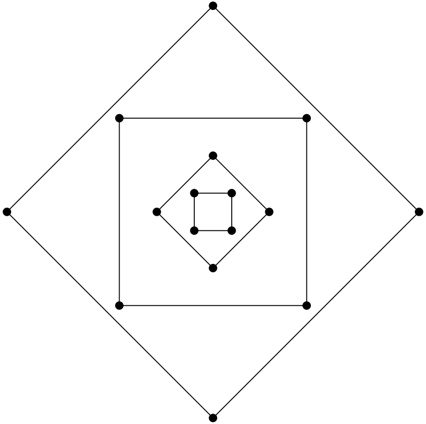
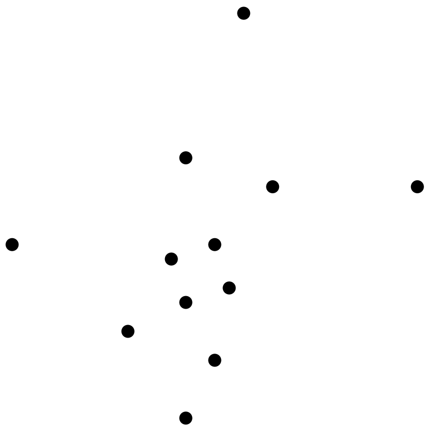


Fish

- Sort the points by the x-axis and by the y-axis.
- In each step, you take the leftmost, topmost, rightmost, and bottommost unmarked points and mark them.
- The last 4 points form the desired smallest rectangle.



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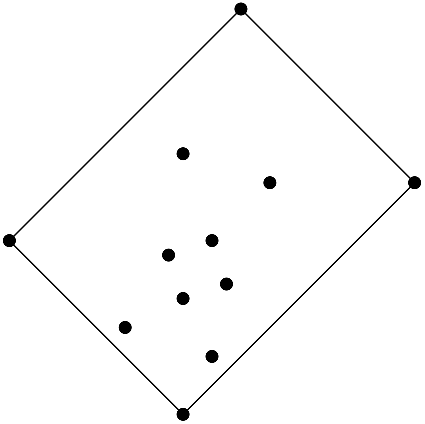

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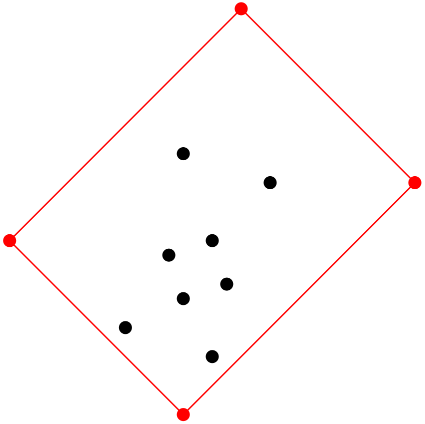

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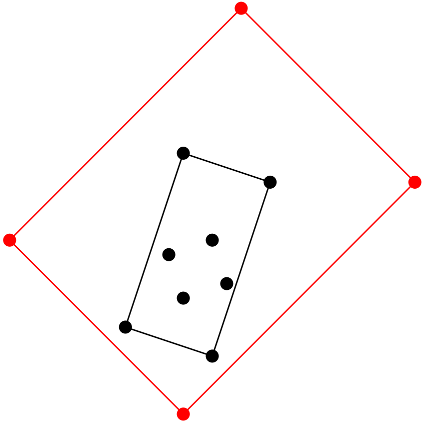

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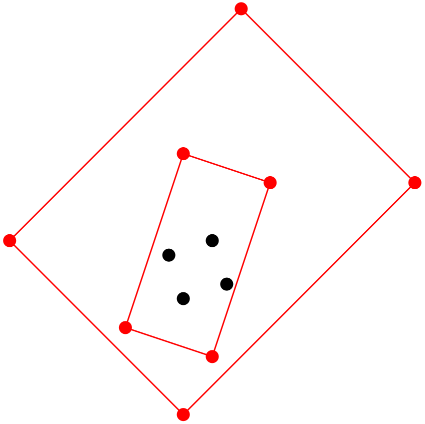

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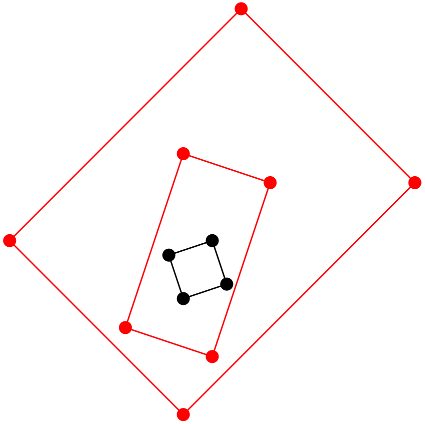
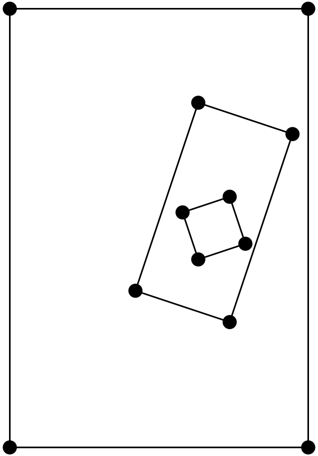


Fish

- It is necessary to take care of the case when a rectangle has edges parallel to the axes.



Ornithology

Ornithology

- **Input**: A bipartite graph with two parts drawn on two parallel straight lines.
- **Output**: The number of edge crossings.
- The edges do not cross at the vertices.

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- Let $V = A \cup B$ and. Edges $\{a_1, b_1\}$ and $\{a_2, b_2\}$ cross if and only if $a_1 < a_2 \wedge b_2 < b_1$ or $a_1 > a_2 \wedge b_2 > b_1$.

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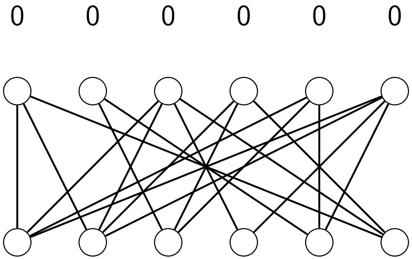
- $O(|E|^2)$ naive algorithm is too slow. ($|E| \approx 2 \cdot 10^5$).

Ornithology - fast solution



res =

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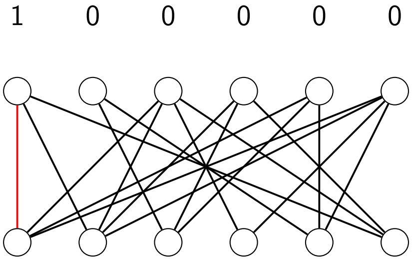

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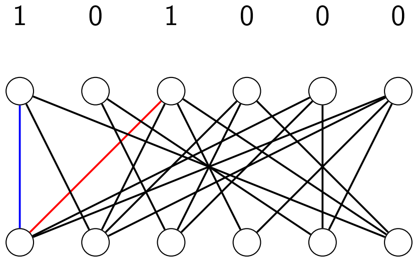

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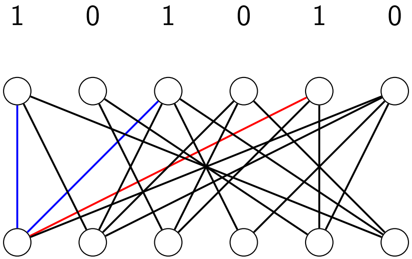

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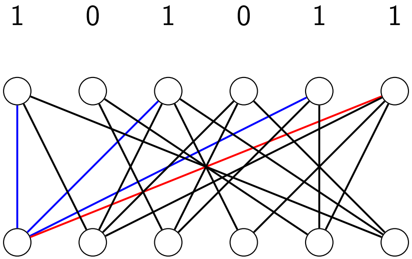

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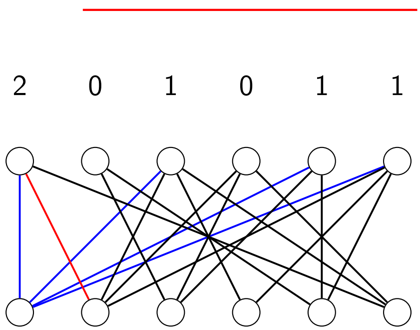
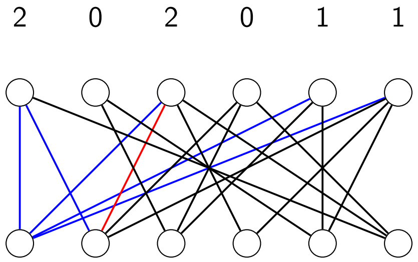


res = 3

Ornithology - fast solution



$$\text{res} = 3 + 2$$

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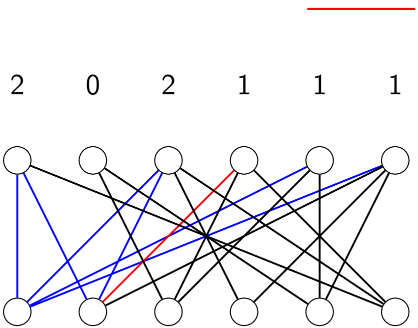

$$\text{res} = 3 + 2 + 2$$

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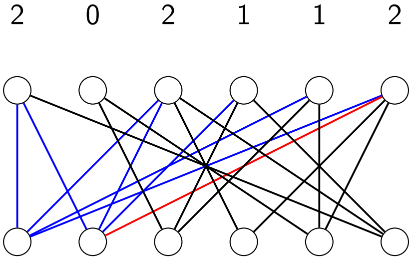

res $= 3 + 2 + 2$

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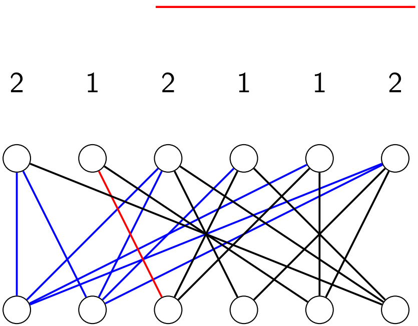

$$\text{res} = 3 + 2 + 2 + 6$$

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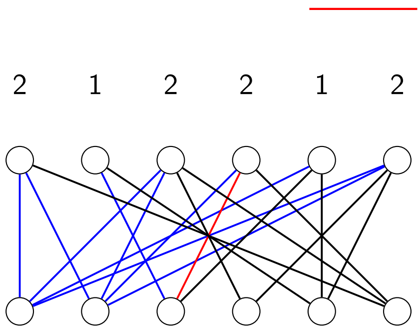

$$\text{res} = 3 + 2 + 2 + 6 + 3$$

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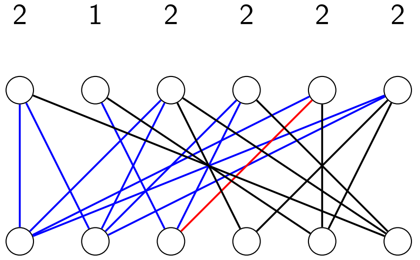

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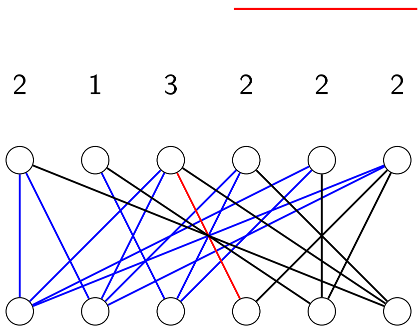

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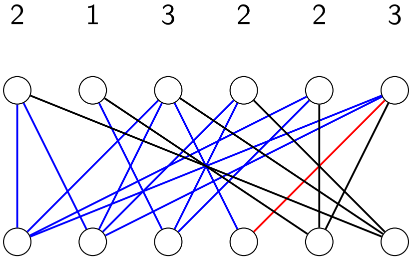

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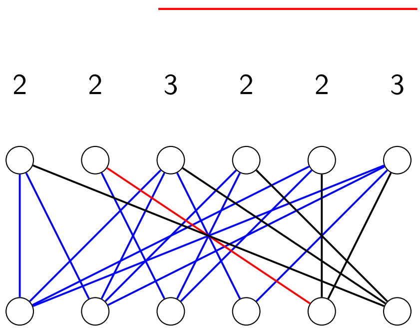

res $= 3 + 2 + 2 + 6 + 3 + 2 + 6 + 10$

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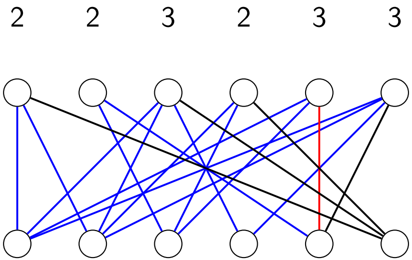

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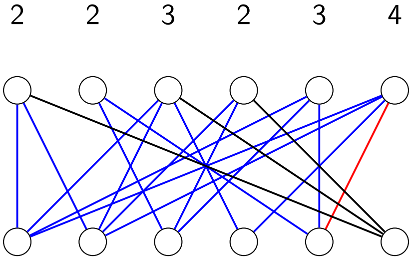
Ornithology - fast solution



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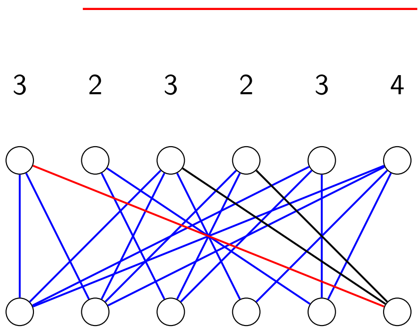

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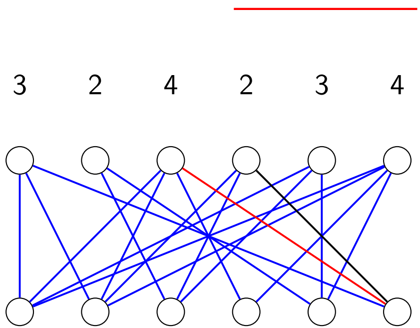

$$\text{res} = 3 + 2 + 2 + 6 + 3 + 2 + 6 + 10 + 3 + 14 + 9$$

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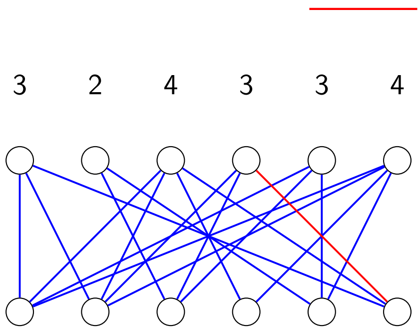

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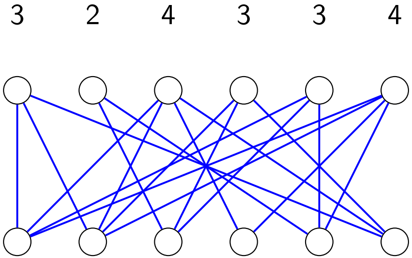

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- Complexity: $O(N\dot{l}og(|U|)\dot{l}og(|U|) + Q\dot{l}og(|U|))$

Watchdogs

Watch~~dogs~~

Watchcats

Watchcats

- **Input:** Tree T with q paths specified by endpoints.
- **Task:** For each a-b path P a vertex x on P with $|d(a,x) - d(b,x)| \leq 1$ must be selected. One vertex can be selected for multiple paths.
- **Output:** Minimum number of vertices to select.

Watchcat Niki

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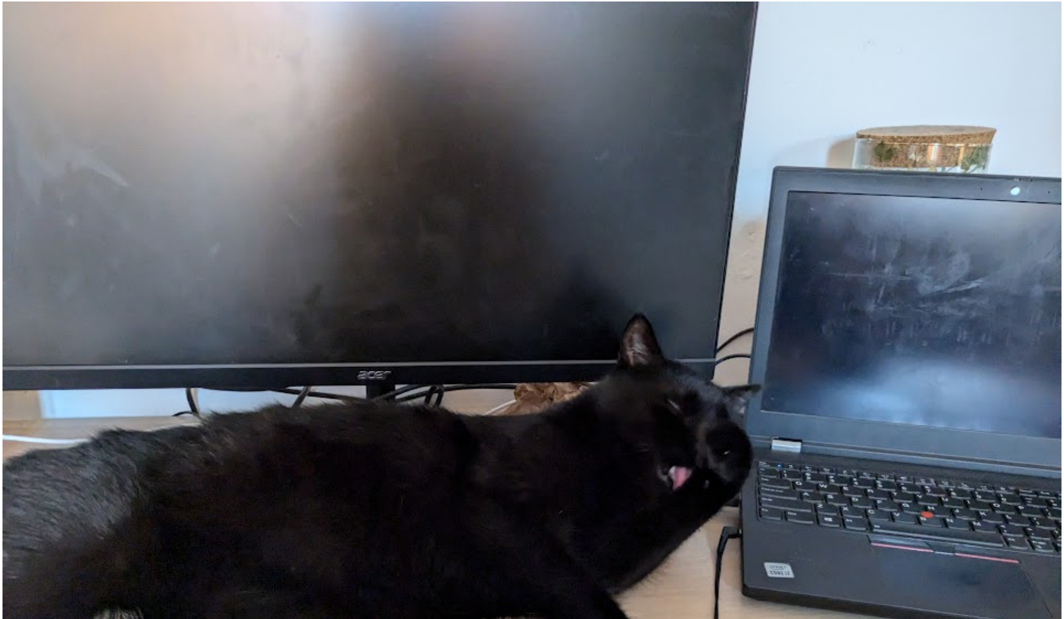
- Vertex Cover on the remaining forest!

Watchcat Čičinas

Watchcat Denis



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- Result is $\min\{D[r][0], D[r][1]\}$.

Watchcat Ritchie

Watchcat Kiwi

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- The case $d(\ell, a) > d(\ell, b)$ is symmetric.
- Total running time: $O(n \log n + q \log n + n) = O((n + q) \log n)$.

Watchcat Jiskra

Cowpproximation

Cowpproximation

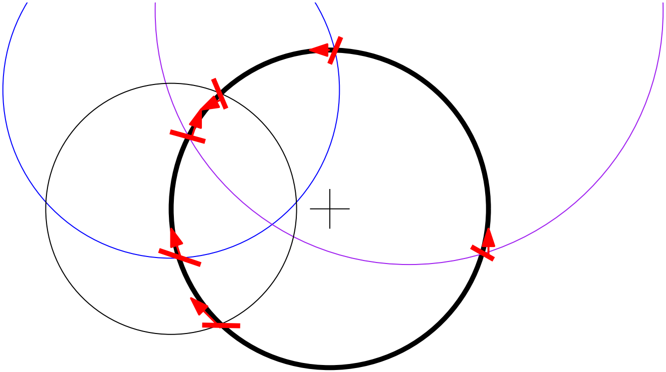
- **Input**: Set of N circles, given by x, y and radius r.
- **Task**: Suppose each circle can travel with speed up to 1 unit of distance per second. In how many seconds can all circles contain a common point?

Cowpproximation

- **Combinatorial solution**:
- **Observation**: We can assume each circle moves at maximum speed and waits at some point if needed.
- Note after t seconds, a circle with radius r can cover exactly the points that are within radius $r + t$.
- We look for minimum t such that if we increase all radii by t, all circles have non-empty intersection.
- Binary search on t.

Cowpproximation

- Verify if the set of circles has nonempty intersection:
- Consider a circle C. If the common intersection contains the boundary of C, we can find it as follows.
- The intersections with C give us intervals on the boundary of C.



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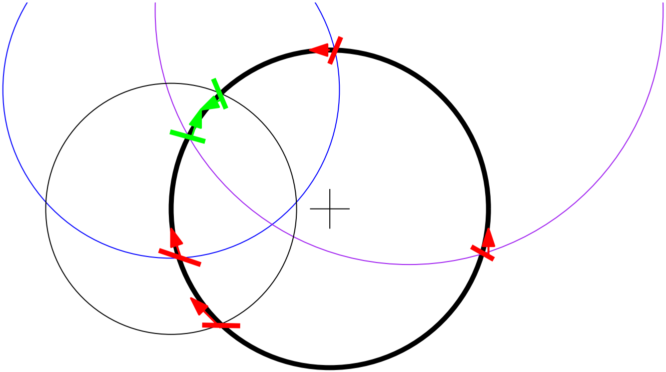

Cowpproximation

- **Observation**: Consider function $f(x, y) = max_{C \in \text{circles}} dist((x, y), C)$.
- The minimum of this function is the solution.
- This function is convex.
- Well implemented gradient descend may find the solution quickly.

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Use dynamic programming!

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- Let Q' be all the possible states q' such that reading character s_i advances A to state q and $1 \leq i \leq |s|$:

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Alternatively, the automaton can be represented implicitly by the regular expression. In such case it is enough to consider a position inside the regex instead of a state in the Automaton.

Pigpartite giraffe

Pigpartite giraffe

- **Input**: Small bipartite graph ($n \leq 8$ vertices in each partite).
- **Queries**: Given vertices v, u, add a new x vertex with neighborhood $N(x) = (N(v) \setminus N(u)) \cup (N(u) \setminus N(v))$.
- **Task**: After each query, compute the total sum of distances within the graph.
- Consider the incidence matrix of the graph. The query corresponds to taking a XOR of two rows (equivalently sum $\mod 2$).

Pigpartite giraffe

- Each vertex can be described by the set of its original ancestors (the original 8 vertices in its partite).
- Only 2^8 possible types of vertices in each partite, hence $2 \cdot 2^8$ different type of vertices in total.
- Two vertices of the same type have the same neighborhood.
- **Observation:** Adding a new vertex of an existing type will not change distances between other vertices.

Pigpartite giraffe

Adjacency matrix of a bipartite graph G looks like

$$\mathbf{G} = \begin{array}{c|cc} & A & B \\ \hline A & \mathbf{0} & \mathbf{M} \\ B & \mathbf{M}^\top & \mathbf{0} \end{array}$$

So $\mathbf{M}$ describes our bipartite graph: rows for partite A, columns for partite B.

$$\mathbf{M}_{v,u} = 1 \iff v \in A, u \in B \text{ have an edge.}$$

Pigpartite giraffe

$$\mathbf{M} = \left(\begin{array}{c|cccc} (1) & 1 & 0 & 0 & 0 \\ (2) & 1 & 1 & 1 & 1 \\ (3) & 0 & 0 & 1 & 0 \\ (4) & 0 & 0 & 0 & 1 \end{array}\right)$$

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Pigpartite giraffe

- How to efficiently compute the distances?
- First compute pair-wise distances and keep the distance matrix D ($O(n^3)$).
- Keep D small by keeping only one vertex for each type.
- Keep the count for each type.
- We will keep at most $d \leq 2 \cdot 2^n$ vertices in D.
- Also keep track of the total distances for each vertex of D.

Pigpartite giraffe

- When adding a new vertex v:
- If v has a **new type**:
  - Compute the distance from v to all others with BFS ($O(d^2)$).
  - Update D for all other vertices: for each pair $x, y \in V$, check if $dist(x, v) + dist(v, y) < dist(x, y)$ ($O(d^2)$).
- If v has an **existing type**:
  - Just update the count of v's type.
  - Sum the distances from each vertex ($O(d)$).
    - Be careful about vertices of the same type as v! Especially if v is just the second vertex of that type.
- A new type appears at most $2 \cdot 2^n$ times.
- Let's evaluate the total runtime.
- $O(n^3 + 2^n d^2 + Qd)$ with $d = O(2^n)$.
- $O(n^3 + 2^{3n} + Q2^n)$ with $n \leq 8, Q \leq 10^5$.
- $\approx 2^9 + 2^{24} + 2^{25}$