

# CERC 2019

Presentation of solutions

December 3, 2019

ABB [ABBA]

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- ▶ Examples: Hash, Z-function, Suffix Arrays, Manacher, Palindromic Trees, ...

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- ▶ The check could be done by multiple string matching algorithms.
- ▶ Examples: Hash, Z-function, Suffix Arrays, Manacher, Palindromic Trees, ...
- ▶ Complexity depends on chosen algorithm:  $\mathcal{O}(N)$  is achievable.

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- ▶ Let  $Q$  be equal to  $\lfloor \sqrt{N} \rfloor$
- ▶ All numbers "lesser" than sqrt:  $\sum_{i=1}^Q \lfloor \frac{N}{i} \rfloor$
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- ▶ There are multiple methods to find the blocks – for example:
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- ▶ Complexity is  $\mathcal{O}(N \log^3 MAX)$

K==S [Kiss]

$K \equiv S$  [Kiss]

- ▶ Task: Find the number of strings of length  $N$  which do not contain any of given strings as a substring.

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- ▶ Observation: It can also be solved by computing the number of strings containing any of them as a substring. Then the solution is  $26^N - \text{ANS}$ .

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- ▶ We have to find out the way to build all such strings.
- ▶ This can be done by constructing the Aho-Corasick automaton.
- ▶ It wasn't needed to construct this automaton efficiently as there are at most  $Q = 100$  characters involved (there will be  $\approx Q$  states in the automaton).

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- ▶ Then we figure out approach to solve the problem, though for much lower limit of  $N$ .
- ▶ We will use Dynamic Programming where the state configuration is combination of length of the string and the state of the automation that we're currently in.
- ▶ Time complexity of this approach is  $\mathcal{O}(N \cdot Q)$ .
- ▶ Too slow to fit the constraints of the problem  $\dots$ .

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- ▶ Finally we need to optimize DP approach to fit in the time limit.
- ▶ We will do so by using Matrix Exponentiation.
- ▶ Similarly to the DP approach we will create the transition matrix and use its  $N$ -th power to compute the answer.
- ▶ Time complexity of this approach is  $\mathcal{O}(L^3 \log N)$ .

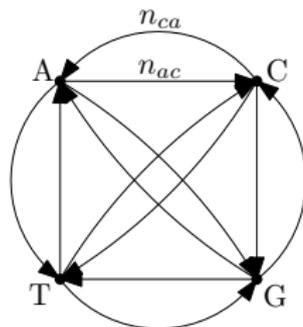
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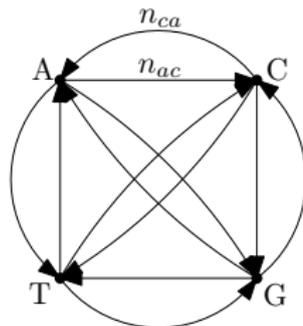
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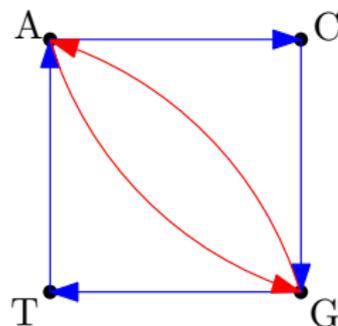
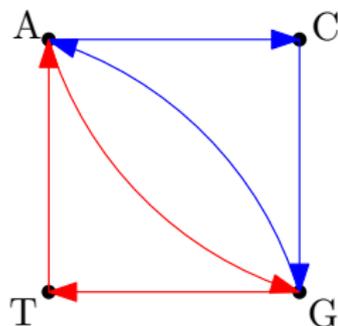
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- ▶ Decompose the graph into maximum possible number of cycles.
- ▶ Greedily start with 2-cycles, then 3-cycles and cover the rest with 4-cycles.
- ▶ Complexity is  $\mathcal{O}(N)$ .

## Ponk Warshall 2-cycles

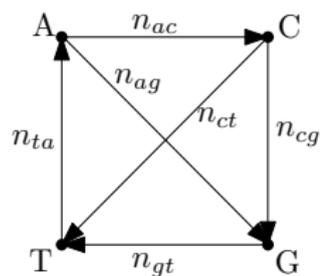
- Assume the 2-cycle is covered differently.



- Then we can change the covering without decreasing the number of cycles.

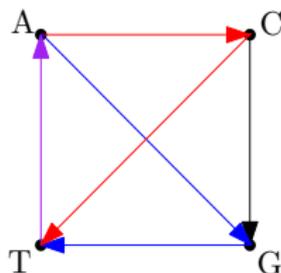
## Ponk Warshall 3-cycles

- ▶ After covering 2-cycles, only one case remains



(up to isomorphism)

- ▶ Only two directed 3-cycles (with one shared edge).



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- ▶ To build the graph we have to consider two cases:
  1. For each query of size  $M$  greater than  $\sqrt{N}$ , go through all edges and check, whether they are in the given set.  
**# of checked edges:**  $\frac{N}{M}N < \frac{N}{\sqrt{N}}N < N\sqrt{N}$
  2. For each query of size  $M$  lesser/equal than  $\sqrt{N}$ , check for all pairs of vertices, whether there is an edge connecting them.  
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**# of checked edges:**  $\frac{N}{M}M^2 = NM \leq N\sqrt{N}$
- ▶ The checking step can be done (for example) with  $\mathcal{O}(\log N)$  overhead if we use some standard set implementation.

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## Saba1000kg [Sabaton]

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- ▶ Complexity is  $\mathcal{O}(N\sqrt{N} \log N)$

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- ▶ Complexity is  $\mathcal{O}(N \log^3 MAX)$

# Screamers in the Storm [by Emerald Sun]

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Thank you for your attention!